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Aerodynamic force reconstruction using physics-informed Gaussian processes

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SUMMARY:
Aerodynamic load models are functions of statistically measurable properties and are derived based on different simplifications of the true physical forces. Validating these models is a complex task, and the effects of the model's assumptions can significantly impact the estimated responses. This paper presents a probabilistic physics-informed machine-learning method to reconstruct the underlying aerodynamic loads based on system measurements. The method is demonstrated using noisy responses of the Great Belt East Bridge due to the application of an aerodynamic load calculated using the linear unsteady model. A very good agreement is observed between the true and regressed loads, and prediction uncertainty is quantified naturally within the machine learning model. The reconstructed loads have a wide range of applications and can be used in practice for modelling validation and system identification.

Keywords: aerodynamic force reconstruction, machine learning, model validation

1. INTRODUCTION

Aerodynamics is a critical field that plays a crucial role in the design and performance of long-span bridges. One of the main challenges in aerodynamics is the accurate prediction of the forces acting on a system, and while several models for this exist, they are generally based on different assumptions and simplifications of the true physical behaviour (Kavrakov and Morgenthal, 2018). In this paper, a physics-informed machine learning approach for force reconstruction is presented, leveraging the knowledge of the system behaviour given by mathematical models to support the training of a Gaussian process regression model (Williams and Rasmussen, 2006). The method overcomes many shortcomings from traditional force reconstruction models (Sanchez and Benaroya, 2014) and offers prediction uncertainty quantification. This study employs the linear unsteady model to evaluate the structural response of the Great Belt East Bridge, which is contaminated with noise and used as input to the force reconstruction model, yielding in turn a stochastic model for the underlying aerodynamic force. Evaluation of the results is carried out by comparing the true and reconstructed signals, and the forces are in good agreement, especially in low-frequency ranges.

2. PHYSICS INFORMED GAUSSIAN PROCESS FOR FORCE RECONSTRUCTION

The dynamic response of a harmonic oscillator given an arbitrary time-varying load F is given by

$$m\ddot{u} + 2m\zeta\omega_n\dot{u} + m\omega_n^2 u = F, \quad (1)$$

where m is the mass, ζ is the damping ratio, ω_n is the circular natural frequency and u, $\dot{u}$ and $\ddot{u}$ are the displacement, velocity and acceleration, respectively. The responses are measurable and, in general, contain different levels of noise based on the quality of the measurement system. The displacements, for instance, can be modelled as $u = f(t) + \varepsilon$, where t is time and $\varepsilon \sim \mathcal{N}(0, \sigma_u^2)$ is the i.i.d. noise with variance σ_u^2, while the velocities and accelerations follow a similar principle.

Assuming the displacement response is a sample from a zero-mean Gaussian process yields

$$u(t) \sim \mathcal{GP}\left(0, k_{uu}(t,t') + \sigma_u^2 \delta(t,t')\right), \tag{2}$$

where δ is the Kronecker-Delta, t and t' are time instants when deflections are measured and k_{uu} is a covariance kernel that reflects the model assumptions. Because the displacement is assumed to be continuous and smooth in time, k_{uu} is modelled herein by the squared exponential (SE) kernel, as given by (Williams and Rasmussen, 2006).

Exploiting the linear time-derivative relation between displacements, velocities and accelerations, covariance kernels can be created to relate different physical measurement types (e.g. $k_{u\dot{u}}$, $k_{u\ddot{u}}$, $k_{\dot{u}\dot{u}}$ or $k_{\ddot{u}\ddot{u}}$), by applying the time derivative in the original GP model from Eq. 2. A summary of this procedure is shown in Fig. 1 (green block). The SE kernel's parameters and the measurement

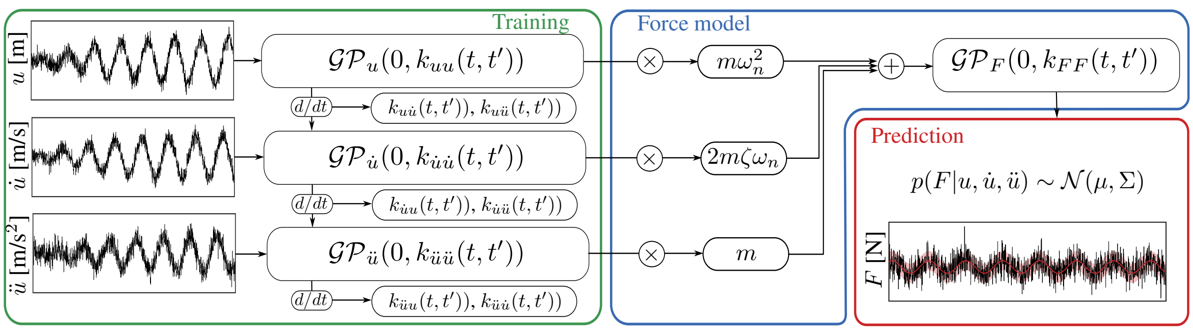

Figure 1. Framework for the physics-informed Gaussian process. Models for different data types are created and jointly trained (green). The force model is built using Eq. 1 (blue) and used for predictions (red).

noise represented by σ_u, $\sigma_{\dot{u}}$ and $\sigma_{\ddot{u}}$ are generally not known a priori, and can be identified from data using maximum likelihood estimation,

$$\boldsymbol{\theta}_{\text{opt}} = \text{argmax}_{\boldsymbol{\theta}} \log p(\boldsymbol{y}|\boldsymbol{t},\boldsymbol{\theta}) = \text{argmax}_{\boldsymbol{\theta}} \left(-\frac{1}{2}\boldsymbol{y}^T \boldsymbol{K}^{-1} \boldsymbol{y} - \frac{1}{2}\log|\boldsymbol{K}| - \frac{N}{2}\log 2\pi \right), \tag{3}$$

where $\boldsymbol{\theta}$ is a vector containing the optimizable parameters, $\boldsymbol{y} = [\boldsymbol{u}, \dot{\boldsymbol{u}}, \ddot{\boldsymbol{u}}]^T \in \mathbb{R}^N$ contains the measurement training data at times $\boldsymbol{t} = [\boldsymbol{t}_u, \boldsymbol{t}_{\dot{u}}, \boldsymbol{t}_{\ddot{u}}]^T \in \mathbb{R}^N$, and $\boldsymbol{K}$ is a covariance matrix calculated as:

$$\boldsymbol{K} = \begin{bmatrix} k_{uu}(\boldsymbol{t},\boldsymbol{t}') + \boldsymbol{I}\sigma_u^2 & k_{u\dot{u}}(\boldsymbol{t},\boldsymbol{t}') & k_{u\ddot{u}}(\boldsymbol{t},\boldsymbol{t}') \\ k_{\dot{u}u}(\boldsymbol{t},\boldsymbol{t}') & k_{\dot{u}\dot{u}}(\boldsymbol{t},\boldsymbol{t}') + \boldsymbol{I}\sigma_{\dot{u}}^2 & k_{\dot{u}\ddot{u}}(\boldsymbol{t},\boldsymbol{t}') \\ k_{\ddot{u}u}(\boldsymbol{t},\boldsymbol{t}') & k_{\ddot{u}\dot{u}}(\boldsymbol{t},\boldsymbol{t}') & k_{\ddot{u}\ddot{u}}(\boldsymbol{t},\boldsymbol{t}') + \boldsymbol{I}\sigma_{\ddot{u}}^2 \end{bmatrix}. \tag{4}$$

With the identified parameters, a joint distribution can be built between the training data in $\boldsymbol{y}$ and the dynamic forces $\boldsymbol{F}$. Conditioning the forces on the data yields

$$p(\boldsymbol{F}|\boldsymbol{y}) = \mathcal{N}(\boldsymbol{\mu} = \boldsymbol{K}_*^T \boldsymbol{K}^{-1} \boldsymbol{y}, \boldsymbol{\Sigma} = \boldsymbol{K}_{**} - \boldsymbol{K}_*^T \boldsymbol{K}^{-1} \boldsymbol{K}_*), \tag{5}$$

where $\boldsymbol{K}_* = [k_{Fu}(\boldsymbol{t}_*, \boldsymbol{t}), k_{F\dot{u}}(\boldsymbol{t}_*, \boldsymbol{t}), k_{F\ddot{u}}(\boldsymbol{t}_*, \boldsymbol{t})]^T$, $\boldsymbol{K}_{**} = k_{FF}(\boldsymbol{t}_*, \boldsymbol{t}_*)$, $\boldsymbol{t}_*$ are the discrete time indexes for force prediction, and k_{Fi} for $i \in \{u, \dot{u}, \ddot{u}, F\}$ are the kernels generated using the harmonic oscillator model given in Eq. 1, following the framework in Fig. 1.

3. AERODYNAMIC ANALYSIS OF THE GREAT BELT EAST BRIDGE

The model developed for force identification is now used to reconstruct the aerodynamic forces applied to a numerical model of the Great Belt East Bridge in Denmark. A sketch of the suspension bridge, along with its cross-section and the coordinate system for the aerodynamic forces are shown in Fig. 2. A reduced order model based on the linear unsteady (LU) (Caracoglia and Jones, 2003) assumption is used to numerically obtain the aerodynamic response of the bridge. Turbulent wind time-histories are generated using the von Karman spectrum, characterized by the turbulence intensities $I_u = 8\%$ and $I_w = 6\%$, and length scales $L_u = L_w = 60$ m. The mean component is assumed as $U = 30$ m/s, and the time histories are correlated in space along the length of the deck. The generated turbulence spectrum for the vertical component w at midspan, and the wind correlation in space are shown in Fig. 3. A total of 22 modes are used in the dynamic analysis of the structure, which is carried out using a time interval of $\Delta t = 0.05$ s. The static wind coefficients and aerodynamic derivatives are obtained from (Kavrakov and Morgenthal, 2018). Buffeting analysis is carried out using Sears' admittance model. Once the responses are obtained, noise with SNR = 20 is added to the true signal and a sparse selection of training points (TPs, cf. Fig. 4, top) at every $\Delta t = 1.25$ s is used to train the GP model. Conditioning the force model on the training data yields a stochastic model for the force, according to Eq. 5, and the mean prediction and 95% confidence interval are shown in Fig. 4, bottom. The force predictions are in very good agreement with the true values calculated by the LU model in the mean sense. The GP outputs are a smoothed version of the original non-linear force. In regions where high-frequency content dominates the forcing signal, the GP model captures the behaviour as part of its uncertainty range. This is further observed in Fig. 4 (bottom, right), where the prediction PSD is compared with the original forcing signal. The regressed force captures the low-frequency content of the LU model force accurately,

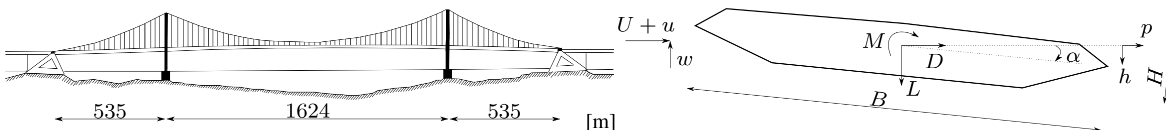

Figure 2. The Great Belt East Bridge in Denmark. Elevation sketch (left) and deck coordinate system (right).

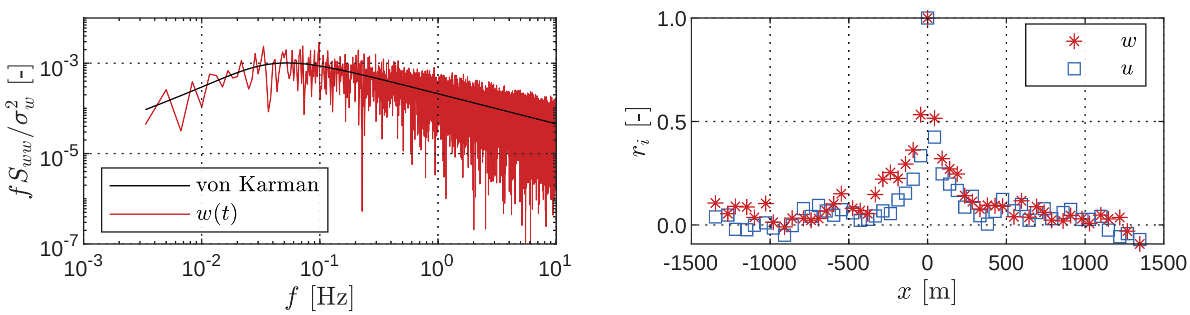

Figure 3. Turbulence spectrum in the vertical direction w (left) and spatial correlation along the bridge length (right).

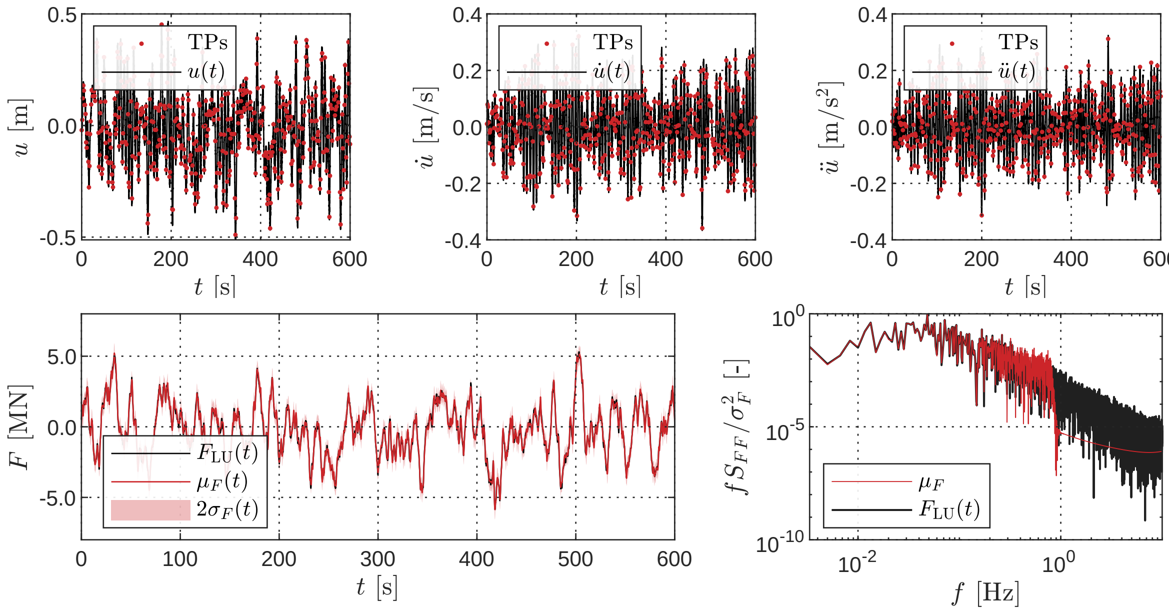

Figure 4. Top: first bending mode responses and training points (TPs). Bottom: true force signal $F(t)$, along with mean and 95% confidence interval of predictions in time (left) and the correspondent power spectral density (right).

and the quality is inversely proportional to increasing frequencies. The predicted force has a very low spectral amplitude for frequencies higher than 0.8 Hz, corresponding to the time interval of the training data. Increasing the time discretisation of the training data, therefore, improves prediction quality and allows the model to capture high frequency content more accurately.

4. SUMMARY AND CONCLUSIONS

In summary, the developed physics-informed machine learning model combines the advantages of artificial intelligence approaches with the structural knowledge given by mathematical models, allowing for the use of heterogeneous and multi-fidelity data during training, and overcoming the need for regularization schemes. Testing was carried out using the linear unsteady model for aerodynamic forces in combination with a numerical model of the Great Belt East Bridge. Although complex non-linear interactions exist between the self-excited and buffeting components and the resultant aerodynamic load, the model is able to accurately regress the underlying force, incorporating high-frequency content and measurement noise into its uncertainty range. The outputs of this analysis can be extended to investigate model accuracy and precision and provide a tool for model comparison, which is an open topic for further investigations.

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